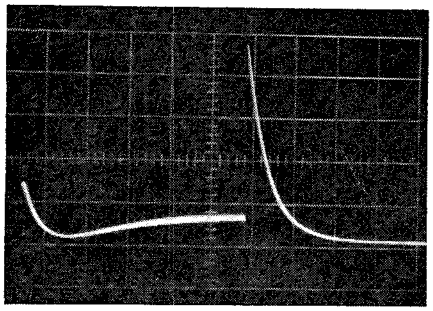

Fig. 3. Reflection of RF pulse from a superconducting cavity. Overcoupled mode, pulse carrier frequency $= f_R = 9375$ Mc/s, pulse duration 110 μs, time scale 20 μs/div. Every relaxation time is read from decay after termination of pulse.

For a sufficiently fast frequency sweep, the amplitude of the transient ringing is small compared to the amplitude of the carrier signal I_0, $m=(I_m/I_0)\ll 1$. Neglecting higher powers of m, one obtains for the detector current

$$i \sim \text{const}\left[1 + 2me^{-(R/2L)t} \cdot \cos\left(\frac{\Delta\omega}{2T}t^2 + \frac{\pi}{4}\right)\right]. \quad (5)$$

Equation 5 shows that the decay of the beat signal is a direct measure of the loaded quality of the cavity

$$\tau_{\text{ampl}} = \frac{2L}{R} = \frac{2Q_L}{\omega_0} = 2\tau.$$

Since a decay proportional to the field amplitude is measured, errors caused by nonquadratic characteristics of the diode are reduced. Also, because of the high sweep rate the measurement is insensitive to spurious FM noise. However, the response time of the demodulator and of the oscilloscope must be fast enough to resolve the beat signal. More precisely, the upper cutoff frequency of the detector must be high compared to the beat frequency when the beat amplitude has decreased by a factor $1/e$, hence,

$$\omega_{\text{cutoff}} > 2\frac{\Delta\omega}{T}\tau.$$

If the response of the receiving system decreases towards higher frequencies, the measured value of the time constant is too small.

The decay time of the beat signal (Fig. 2) has been compared with other nonequilibrium measurements using RF pulses with rise and fall times which are short compared to τ [5]. Figure 3 shows the time behaviour of RF pulses after reflection from the cavity. The carrier frequency has been adjusted to the resonant frequency of the cavity. The measured relaxation time after termination of the pulse is 10 μs, compared to an amplitude decay time of 20 μs observed with rapid frequency alteration and response. The loaded Q of the resonator in the superconducting state is $Q_L = \omega_R\tau \sim 5.9 \cdot 10^5$ for the strongly excited resonance (overcritically coupled) and $Q_L \sim 6.3 \cdot 10^6$ for the weakly excited quasi-degenerate mode (undercritically coupled). The half-power widths calculated from the decay times are $\Delta f = 15.9$ kc/s and $\Delta f = 1.5$ kc/s, respectively, in fair agreement with the values of Δf extrapolated from the quasi-static resonance curve.

It is interesting to note that with a sufficiently fast frequency modulation the beat phenomenon can be observed in any conventional microwave cavity. Calculations show that with an increasing sweep rate the resonance curve first becomes asymmetrical with a smooth initial flank and a gradually increasing overshoot. The apparent width of the resonance curve is increased, i.e., with a fast sweep the Q value determined from the half-power points is too low.

H. J. Schmitt
H. Zimmer
Philips Zentrallaboratorium GmbH
Hamburg, Germany

References

[1] A. F. Harvey, *Microwave Engineering*. New York: Academic, 1963, pp. 202–204.
[2] S. O. Rice, "Statistical properties of a sine wave plus random noise," *B.S.T.J.*, vol. 27, pp. 109–157, January 1948.
[3] W. G. Cady, *Piezoelectricity*, vol. 1. New York: Dover, 1964, pp. 385–387.
[4] A. Erdelyi, *Asymptotic Expansions*. New York: Dover, 1956.
[5] H. Zimmer, "Ein supraleitender Resonator hoher Güte bei 9375 MHz," *Philips: Unsere Forschung in Deutschland*, Philips Zentrallaboratorium, GmbH, Aachen, Hamburg, 1964, pp. 134–136.

Re-Entrant Directional Couplers Having Direct Coupled Center Conductors

The re-entrant coaxial coupler [1] and its strip transmission line equivalent [2] are shown in Figs. 1 and 2, respectively. In both configurations, no direct coupling between center conductors exist. The purpose of this communication is to point out that if direct coupling between center conductors is permitted, the required value of Z_{01} is reduced [1] (see Figs. 1 and 2), thus often permitting easier realization of the coupler than would otherwise be the case.

For both the networks of Figs. 1 and 2, the capacitance network at an arbitrary cross section of the re-entrant coupler with direct coupling between center conductors is given in Fig. 3. The direct capacitance between center conductors is C_{12}. The capacitance between one center conductor and the floating shield is C_{N1}. The capacitance between the shield and ground is C_{NG}.

The design equations for the network of Fig. 3 are

$$376.7\,Y_{oo}/\sqrt{\epsilon_r} = C_{N1}/\epsilon + 2C_{12}/\epsilon \quad (1)$$

and

$$\frac{\sqrt{\epsilon_r}\,Z_{oe}}{376.7} = (C_{N1}/\epsilon)^{-1} + 2(C_{NG}/\epsilon)^{-1} \quad (2)$$

Manuscript received December 8, 1965.

where:

Z_{oe} = the required even-mode impedance of the directional coupler [3]
Y_{oo} = the required odd-mode admittance of the directional coupler [3]
ϵ_r = the relative dielectric constant of the medium, and
ϵ = the permittivity of the medium.

In general,

$$C/\epsilon = \frac{376.7}{\sqrt{\epsilon_r}\,Z} = \frac{376.7\,Y}{\sqrt{\epsilon_r}}. \quad (3)$$

Note that for $C_{12}=0$ (1) and (2) reduce to (1) and (2) of Cohn [1].

The normalized capacitance C_{NG}/ϵ is determined from the network of Fig. 4 and by (3). In this case the center conductors are driven in the even-mode [3]. The normalized capacitances C_{12}/ϵ and C_{N1}/ϵ are determined from the network of Fig. 5. In this case

$$C_{N1}/\epsilon = 376.7\,Y_{oe}/\sqrt{\epsilon_r} \quad (4)$$

and

$$C_{12}/\epsilon = \frac{376.7}{\sqrt{\epsilon_r}}\,\frac{(Y_{oo} - Y_{oe})}{2}, \quad (5)$$

where Y_{oo} and Y_{oe} are the odd- and even-mode admittances [3], respectively, of the network of Fig. 5.

A short numerical example will show how direct coupling between center conductors reduces the required Z_{01}. Let it be required to design a -3.0 ± 0.4-dB directional coupler over a 1.93-to-1 frequency band. Let the relative dielectric constant of the medium be 2.3. The required coefficient of coupling is

$$k^2 = 0.55. \quad (6)$$

It is determined from design equations given in [3] that

$$\sqrt{\epsilon_r}Z_{oe} = 197.7 \quad (7)$$

and

$$\sqrt{\epsilon_r}Z_{oo} = 29.33. \quad (8)$$

From (1) and (2), the required normalized capacitances satisfy

$$0.5248 = (C_{N1}/\epsilon)^{-1} + 2(C_{NG}/\epsilon)^{-1} \quad (9)$$

and

$$12.84 = C_{N1}/\epsilon + 2C_{12}/\epsilon. \quad (10)$$

Case 1: $C_{12}/\epsilon = 0$. With $C_{12}=0$, (9) and (10) may be solved directly.

$$C_{N1}/\epsilon = 12.84 \quad (11)$$

and

$$C_{NG}/\epsilon = 4.475. \quad (12)$$

Equation (12) corresponds to

$$\sqrt{\epsilon_r}\,Z_{01} = 84.2, \quad (13)$$

where we recall that Z_{01} is determined from the network of Fig. 4.

Case 2: $C_{12}/\epsilon = 1.5$. Then

$$C_{N1}/\epsilon = 9.843, \quad (14)$$

and

$$C_{NG}/\epsilon = 4.726. \quad (15)$$

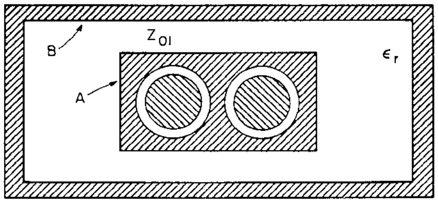

Fig. 1 Re-entrant coaxial directional coupler (Z_{01} is the characteristic impedance of the coaxial line with center conductor A and outer conductor B).

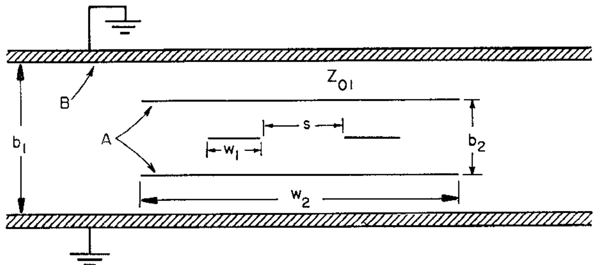

Fig. 2. Strip transmission line re-entrant directional coupler. (Z_{01} is the characteristic impedance of the coaxial line with center conductor A and outer conductor B) (Constraint: $w_2 \geq 2w_1 + s + 2b_2$.)

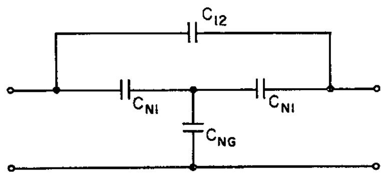

Fig. 3. Capacitance network at an arbitrary cross section of a re-entrant coupler with direct coupling between center conductors.

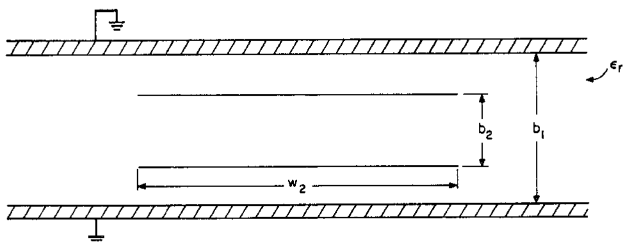

Fig. 4. Network for determination of C_{NG} and Z_{01}.

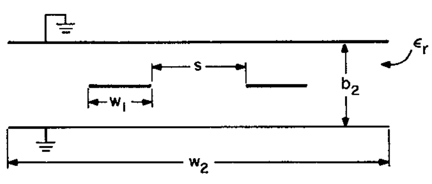

Fig. 5. Network for determination of C_{N1} and C_{12}.

Equation (15) corresponds to

$$\sqrt{\epsilon_r} Z_{01} = 79.7. \qquad (16)$$

This is a 5.31-percent reduction in the required Z_{01}. In a similar calculation, a value of $C_{12}/\epsilon = 3$ gave a 15.3-percent reduction in the required Z_{01}.

Conclusion

Directional couplers of re-entrant cross-sectional and direct coupled center conductors give smaller required values of Z_{01} and, therefore, may often permit easier realization of Z_{01} than similar couplers without direct coupled center conductors.

Edward G. Cristal
Stanford Research Institute
Menlo Park, Calif.

References

[1] S. B. Cohn, "Re-entrant cross-section and wide-band 3-dB hybrid coupler," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-11, pp. 254–258, July 1963.

[2] L. Lavendol and J. J. Taub, "Re-entrant directional coupler using strip transmission line," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-13, pp. 700–701, September 1965.

[3] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 75–81, April 1956.

On Incidental Dissipation in High-Pass and Band-Stop Filters

For narrow-band microwave band-stop filters, Young, et al.[1] have obtained valuable expressions for the effects of incidental dissipation. The purpose of this correspondence is to remark that for the uniformly dissipative (i.e., equal Q) case, the response of the lossy low-pass prototype (and hence, the response of the lossy band-stop filter) may be obtained from the lossless response by means of the frequency transformation.

$$p = \frac{s}{1 + ds} \qquad (1)$$

where

$$d = \frac{1}{wQ}$$

and p and s are the complex frequency variables for the lossless and lossy cases, respectively. Also, w is the normalized pass band-width defined by Young, et al.,[1] and Q is the quality factor of each resonator of the band-stop filter.

To obtain (1), it is merely necessary to observe that each lossy resonator of the band-stop filter (Fig. 8 of Young et al.[1]) corresponds, in the low-pass prototype, to either an inductor and a resistor in parallel or a capacitor and a resistor in series, and then to carry out the details of the frequency transformation.

For estimation of pass band losses, (1) may be put in more convenient form. From (1), obtain

$$= \frac{s - ds^2}{1 - (ds)^2}$$

and, for $s = j\omega$,

$$p = \frac{d\omega^2 + j\omega}{1 + (d\omega)^2}. \qquad (2)$$

For $d\omega \ll 1$

$$p \approx d\omega^2 + j\omega. \qquad (3)$$

Thus, at frequencies for which (3) is valid, the lossy low-pass prototype may be considered to have a frequency-dependent dissipation factor $\delta = d\omega^2$. It follows that the loss may be computed to first order from the group delay according to the well-known relation[2], which becomes

$$L_d(\omega) = A_0(\omega) + 8.686 d\omega^2 T_g(\omega). \qquad (4)$$

Here, $A_0(\omega)$ and $T_g(\omega)$ are, respectively, the attenuation and group delay of the lossless low-pass prototype, while L_d is the attenuation (decibels) of the lossy low-pass prototype.

Equation (4) is not exact, due to both approximation (3) and the error involved in truncating the Taylor series expression for L_d, but for small d, these errors should not be large. For the equal Q case only, then, (4)

Manuscript received December 8, 1965.

[1] L. Young, G. L. Matthaei, and E. M. T. Jones, "Microwave band-stop filters with narrow stop bands," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 416–427, November 1962.

[2] H. W. Bode, *Network Analysis and Feedback Amplifier Design*. Princeton, N. J.: Van Nostrand, 1945, pp. 216–222.